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THE ANALYSIS OF THE STABILITY OF SHOCK WAVES*

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When studying the stability of shock waves in arbitrary media, D'yakov /1/ obtained a dispersion equation which always contains a root with $\operatorname{Im}\omega > 0$, corresponding to the presence of an exponentially increasing multiplier. This fact was mentioned in /2/, although without any explanation. The dispersion equation of /1/ was quoted in /3, 4, 5/, again without any explanation. The purpose of the present paper is to discover why the presence of such a root in the problem in question does not lead to instability of the wave, i.e. why it is of secondary importance in the problem of stability.

The solution for the perturbations behind the shock wave was sought in /1/ in the form $f = \gamma \exp i(kx + ly - \omega t)$, where the y axis is directed along the normal to the wave, the x axis is directed along the wave, $f = \{\delta s, \delta v_x\ \delta v_y \delta p\}$ is the column vector for the perturbed quantities and the wave numbers k, l are connected with the frequency ω by the relation

$$[(\omega - vl)^2 - c^2(k^2 + l^2)](\omega - vl)^2 = 0 \tag{1}$$

Here v, c is the unperturbed velocity of the gas and the speed of sound behind the wave.

For given ω and k, Eq.(1) represents a characteristic equation for the system of ordinary differential equations describing the variation in the perturbations along the y axis. Its solution

$$vl_1 = \omega$$
$$vl_{2,3} = [-M^2\omega \pm \sqrt{M^2(\omega^2 + k^2v^2) - v^2k^2}](1 - M^2)^{-1} \tag{2}$$

determines entropically the vortical wave (the first relation) and two sonic waves (the second relation). Since the source of the perturbations at infinity does not exist, it follows that we seek the solution of the boundary-value problem in the form of a superposition of the waves moving away from the front $\|f_j\| = \|\gamma_{jm}\| \cdot \|C_m e^{i\lambda_m y}\|$ (in the present case $m = 1, 2$). To find C_m, $\|f_l\|$ we substitute, in the linear homogeneous boundary condition at the front for $y = 0$

$$\|B_{lj}\| \cdot \|\gamma_{jm}\| \cdot \|C_m\| \equiv \|A_{lm}\| \cdot \|C_m\| = 0$$

The requirement that a non-trivial solution exist yields the dispersion relation $|A_{lm}| = 0$ of the form /1/

$$2\omega\frac{v}{v_0}(k^2v^2 + \omega^2) = \left(\omega^2\frac{v}{v_0} + k^2v^2\right)[\omega + l_2(\omega, k)]\left[1 + j^2\left(\frac{\partial V}{\partial p}\right)_n\right] \tag{3}$$

Here v_0 is the velocity of gas before the wave, j is the mass flux across the discontinuity, V and p are the specific volume and pressure behind the front, the index "H" denotes differentiation along the adiabatic, and $l_2(\omega, k)$ is the root from (2) with a plus sign (diverging). When the shock wave is stable, $\operatorname{Im}\omega < 0$. From (2) it follows that when $\omega = \pm ikv$, $l_2 = \omega/v$ and $\omega = \pm ikv$ is a root of Eq.(3). Thus the dispersion relation has a root corresponding to an exponentially increasing perturbation.

However, we can easily see that $l_2 = \omega/v$ becomes a triple root corresponding to the case in which the entropically-vortical and diverging sonic pertubations are identical. Unlike the root which is always multiple and corresponds to an entropically-vortical perturbation, $l_2 = \omega/v$ has a non-simple elementary divider of the matrix $\|\mathbf{P}\|$ of the equation $d\mathbf{f}/dy = f\mathbf{P}$. In this case the solutions $\{\gamma_{lm} e^{i\lambda_m y}\}$ are not linearly independent /6/, i.e. they do not form a fundamental system of solutions. From this it follows that the matrix $\|A_{jm}\|$ defining the system of equations for determining C_m, is degenerate, i.e. the dispersion equation $|A_{lm}| = 0$ holds automatically. In the case of a non-simple elementary divider the solution of the system has the form $f = (\gamma_{j1}y + \gamma_{j2})e^{il_2 y}$.

It can be shown that the determinant of the system for γ_j, following from the boundary condition on the shock wave is not zero, i.e. only the trivial solution $\gamma_{lm} = 0$ $(m = 1, 2)$ exist. Therefore, the values $\omega = \pm ikv$, $l_2 = \omega/v = ik$ do not produce increasing perturbations although formally they represent a solution of Eq.(3).

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Thus the presence of the roots with $\mathrm{Im}\,\omega > 0$ in the dispersion equation may not lead to instability, provided that it corresponds to the multiple root of the characteristic equation with a non-simple elementary divider of the corresponding matrix. It is important to keep this in mind when studying the dispersion equation using the argument principle.

During the further analysis of stability in /1/, the dispersion Eq.(3) was reduced, without explanation, by the multiplier $(\omega^2 + k^2v^2)$, corresponding to the root in question. A dispersion equation was obtained in /2/ not containing the roots ω, corresponding to the multiplier roots of Eq.(1), since another method was used which was based on the use of the Laplace transform to represent the solution.

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